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Exact Solution Using Sextic–Quintic Equations for Rectangular and Circular Footing Design

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ABSTRACT

The determination of footing dimensions subjected to axial load and bending moment is commonly performed using a trial-and-error procedure in conventional structural design. This approach often requires iterative calculations and may lead to inefficient design processes. This study presents an analytical formulation for determining the dimensions of rectangular and circular footings by transforming the governing design equations into polynomial forms. The derivation results in sextic equations for rectangular footings and quintic equations for circular footings. These nonlinear polynomial equations are solved using numerical solution techniques to obtain the exact footing dimensions that satisfy the structural design requirements. The proposed formulation eliminates the need for repetitive trial calculations and provides a systematic computational approach for footing design under combined loading conditions. The results demonstrate that the developed analytical–numerical approach can efficiently determine the required footing dimensions while maintaining the design safety criteria. This method provides an alternative computational tool that may assist engineers in performing more accurate and efficient footing design analysis.

Key word: Footing design, Numerical analysis, Sextic equation, Quintic equation, and Structural foundation.

INTRODUCTION

Footing foundations are widely used in structural engineering to transfer loads from superstructures to the supporting soil [1]. In practical applications, footings are frequently subjected not only to axial loads but also to bending moments caused by eccentric loading or lateral forces. Therefore, the design of footing dimensions must ensure adequate load distribution and structural safety while satisfying the bearing capacity requirements of the supporting soil [2]. In conventional structural design practice, the determination of footing dimensions is commonly performed using a trial-and-error procedure. Engineers typically assume an initial footing dimension and then verify whether the design satisfies the required structural and geotechnical criteria. If the assumed dimension does not meet the design requirements, the calculation must be repeated until a satisfactory solution is obtained. Although this approach is widely used in engineering practice, it may result in inefficient calculations and does not provide a direct analytical relationship between design parameters and the resulting footing dimensions [3].

Several studies in structural and foundation engineering have attempted to improve the footing design procedure by introducing analytical formulations and computational methods. However, many of these approaches still rely on iterative procedures or simplified assumptions, which may limit the accuracy and efficiency of the design process, particularly when dealing with combined loading conditions such as axial load and bending moment. Previous studies and literatures [4-8] provides scientific correlations between eccentricity (related to working pressure), punching shear, and the footing's dimension separately, indicating the need to iterate the value of footing dimension to be checked with pressure and punching shear criteria.

To address these limitations, this study proposes an analytical formulation for determining the dimensions of rectangular and circular footings subjected to combined axial load and bending moment. This study provides direct combination of the correlation between material properties, eccentricity, punching shear, and the dimension of the footings. The governing design equations are transformed into polynomial forms, resulting in sextic equations for rectangular footings and quintic equations for circular footings. These nonlinear polynomial equations are then solved using numerical solution techniques to determine the exact footing dimensions that satisfy the structural design requirements.

The proposed analytical–numerical approach provides a systematic computational framework for footing design without relying on repetitive trial-and-error procedures. By employing sextic and quintic polynomial equations, the method enables a direct determination of footing dimensions under combined loading conditions. This approach is expected to improve the efficiency and accuracy of footing design analysis and provide an alternative computational tool for structural and foundation engineers. Therefore, the objective of this study is to develop an analytical–numerical formulation for determining rectangular and circular footing dimensions using sextic and quintic polynomial equations under combined axial load and bending moment.

RESEARCH METHODS

Materials

In foundation design, the determination of footing foundation's dimensions, *Bp* (foundation's width) and *Lp* (foundation's length) are determined by Bearing Capacity, working load, and punching shear criteria. Meanwhile the value of h (foundation's height) is predetermined value based on trial and error (Figure 1.). According [9],[10], the minimum thickness of edge should not be less than 150 mm, without calculation. When h insufficient then the dimension of h will enlarge, however when h meets the condition, rarely checked whether h excessive or not, so as to be generally can occur too overuse of correlates with the cost. The purpose of this study is to determine the need for checking of the effectiveness from a higher beam.

Axial load and moments transferred from the foundation will produce certain amount of pressure working on the soil. In relation to eccentricity, the dimension of the footing will affect the value of the pressure produced, which are compared with soil bearing capacity [11]. However, in this research bearing capacity will not be considered, in other words the external load (axial load and bending moments) that raises land tension smaller than the capacity, so the load review must be smaller than the capacity, and it is assumed that the soil below has infinite value of rigidity and shear strength [18-20].

The material properties, column dimensions, and the working load (axial load and bidirectional bending moments) will be analyzed to determine footing foundation's height, along with its width and length. The methods were used is analyze the determination of foundation's height, use sextic equation or quintic equation which will be produced as many as six values or five values of different heights. From those six values or five values will be filtered with provisions of the results obtained shall a real number, with positive marked > 0 and it valued with smaller of positive results [8] not specified to define d and [12] d by trial:

$$d = \frac{q(L/2 - b/2)}{\tau_c + q} \tag{1}$$

$$q_{u(R)} = q_{u(UR)} + \frac{2c_a d}{B} + \gamma d^2 \left(1 + \frac{D_f}{d}\right)\frac{K_s \tan\varphi}{B} - \gamma d + \sum_{i=1}^{N}\left(\frac{2T_i}{B}\right) \tag{2}$$

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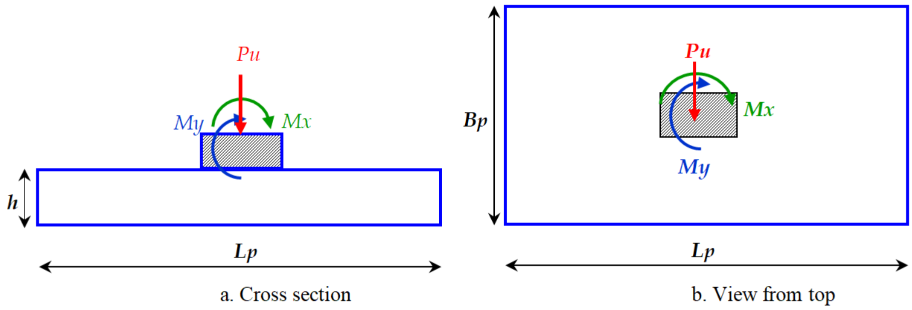

Figure 1. Footing

The foundation's height reviewed in this research based on [9], the coefficient of Ø = 0.75, the thickness of concrete's cover relating to land is 75 mm, one-way act like as a beam, and two-way act. Meanwhile a solution to solving the sextic equation, the most accurate is using Bairstow Method [13].

One way punching shear

Critical cross-section for the footing foundation's shear assumed extends across the entire width and located at a distance of d from the centralized of concrete's surface. Figure 2 shows one way punching shear [3]. The shear forces nominal cross-section if only shear and flexible work, according to [9],[14],[15].

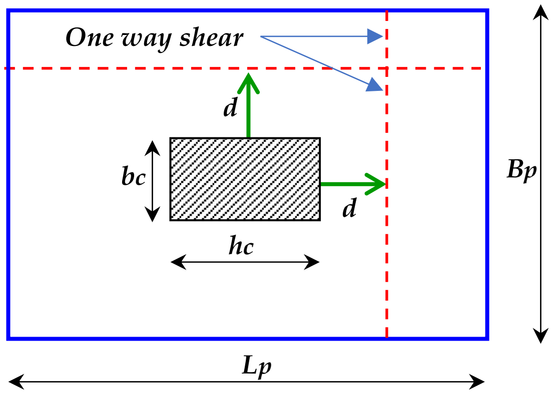

Figure 2. One way punching shear

on Figure 2, direction x, bw = Lp ; direction y , bw = Bp

$$V_c = 0.17\lambda\sqrt{f'_c}\, b_o d \quad (3)$$

Where : $b_o = MIN(Lp\,; Bp)$

Two way punching shear

Cross-section critical perpendicular a slab assumed have the minimum perimeter b_0 ,. Figure 3, showing the position of two-way punching shear [14], [17].

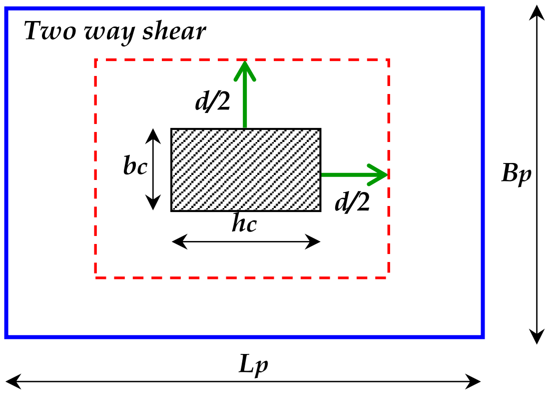

Figure 3. Two way punching shear

Cross-section critical is not need closer than d/2 to the natural axis. According to [9], the forces nominal maximum allowed is the smallest from three results equation (4.a), (4.b) and (4.c),

$$V_c = 0.17\left(1 + \frac{2}{\beta}\right)\lambda\sqrt{f'_c}\,b_o d \tag{4.a}$$

$$V_c = 0.083\left(\frac{\alpha_s d}{b_o} + 2\right)\lambda\sqrt{f'_c}\,b_o d \tag{4.b}$$

$$V_c = 0.33\lambda\sqrt{f'_c}\,b_o d \tag{4.c}$$

Which , [16],[17]:

β = long sides of cl short sides of cs burden centered or reaction area

bo = $2(b_c + d + h_c + d)$, critical cross section perimeter, which is, the failure of idealizing length

αs = 40 for internal column, 30 for external column, 20 for corner column

λ = 1

Dimension of Rectangular Footing

Dimension of footing are *Bp, Lp, d*, dimension of column *bc, hc*, Axial load and biaxial bending moment, *Pu, Mx, My*, (fig.4, 5).

$$e_y = \frac{M_x}{P_u} \tag{5.a}$$

$$e_x = \frac{M_y}{P_u} \tag{5.b}$$

$$P_u = B_p L_p q \tag{6}$$

$$Bp = bc + 4d + 2\,d' \tag{7.a}$$

$$Lp = hc + 4d + 2\,d' \tag{7.b}$$

$$A_c = h_c b_c \tag{8}$$

$$s_k = h_c + b_c \tag{9}$$

$$A_p = B_p L_p$$

$$A_p = (h_c + 4d + 2d')(b_c + 4d + 2d') \tag{10}$$

According to [14],

$$P_u = 2dv_c(b_c + d) + 2dv_c(h_c + d) + (b_c + d)(h_c + d)q \tag{11}$$

Subtitute Eq (6) to Eq(11), become

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$$d^2(4v_c + q) + d(2v_c + q)s_k - (A_p - A_c)q = 0 \quad (12)$$

$$q = \frac{P_u}{A_p} \pm \frac{M_x}{W_x} \pm \frac{M_y}{W_y} \quad (13)$$

$$W_x = \frac{L_p B_p^{\ 2}}{6} \quad (14.a)$$

$$W_y = \frac{B_p L_p^{\ 2}}{6} \quad (14.b)$$

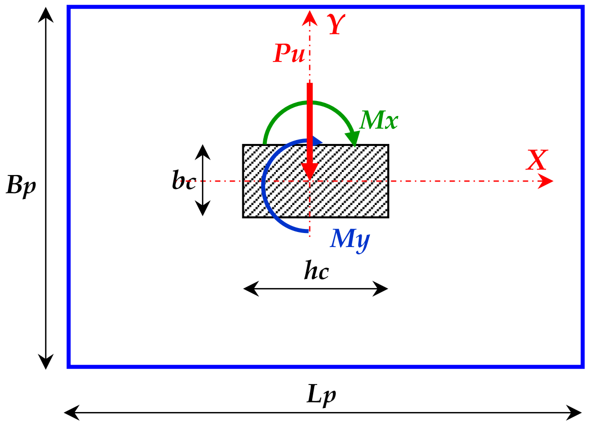
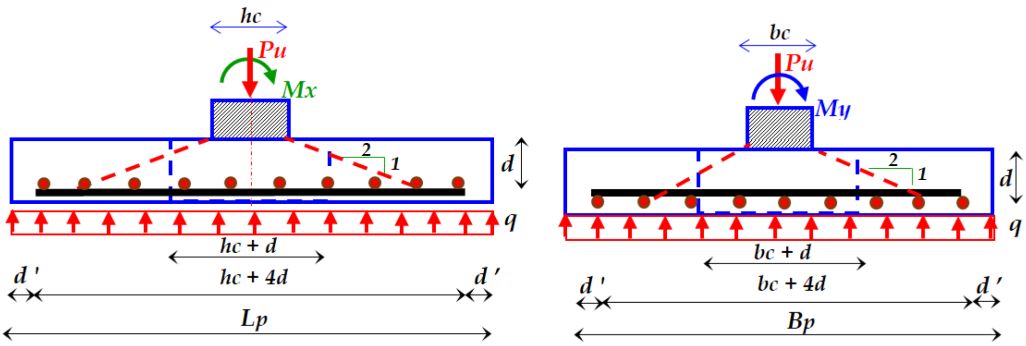


Figure 4. Axial Load Pu, Moment Mx dan My



a. Y axis b. X axis

Figure 5. Axial Load Pu, Mx, My (X axis)

where : *d'* is minimum concrete's cover ≥ 75 mm [9]
Material properties of footing, *fc'*.
vc = maximum from equation (3) and (4)

$$v_c = 0.33\sqrt{f'_c} \quad (15)$$

Subtitute (13) to (12),

$$d^2\left(4v_c + \frac{P_u}{A_p} + \frac{M_x}{W_x} + \frac{M_y}{W_y}\right) + d\left(2v_c + \frac{P_u}{A_p} + \frac{M_x}{W_x} + \frac{M_y}{W_y}\right)s_k - (A_p - A_c)\left(\frac{P_u}{A_p} + \frac{M_x}{W_x} + \frac{M_y}{W_y}\right) = 0 \quad (16)$$

And subtitute (14.a) and (14.b) to (16),

$$d^2\left(4v_c + \frac{P_u}{A_p} + \frac{6M_x}{A_p B_p} + \frac{6M_y}{A_p L_p}\right) + d\left(2v_c + \frac{P_u}{A_p} + \frac{6M_x}{A_p B_p} + \frac{6M_y}{A_p L_p}\right)s_k - (A_p - A_c)\left(\frac{P_u}{A_p} + \frac{6M_x}{A_p B_p} + \frac{6M_y}{A_p L_p}\right) = 0 \quad (17)$$

Eq(17) × A_p^2, become

$$d^2\left(4v_c A_p^2 + P_u A_p + 6M_xL_p + 6M_yB_p\right) + d\left(2v_cA_p^2 + P_u A_p + 6M_xL_p + 6M_yB_p\right)\left(h_c + b_c\right) - \left(A_p - b_ch_c\right)\left(P_u A_p + 6M_xL_p + 6M_yB_p\right) = 0 \quad (18)$$

Subtitute (5.a), (5.b), (7.a), (7.b), (10) to (18),

$$Ad^6 + Bd^5 + Cd^4 + Dd^3 + Ed^2 + Fd + G = 0 \quad (19)$$

where :

$$A = 1024\ vc \quad (20)$$

$$B = (1024\ b + 2048\ d' + 1024\ h)\ vc \quad (21)$$

$$C = -240\ Pu + (320\ b^2 + 1792\ b\ d' + 1536\ d'^2 + 768\ b\ h + 1792\ d'\ h + 320\ h^2)\ vc \quad (22)$$

$$D = -360\ Mx - 360\ My + (-108\ b - 496\ d' - 108\ h)\ Pu + (32\ b^3 + 448\ b^2\ d' + 1152\ b\ d'^2 + 512\ d'^3 + 192\ b^2\ h + 1024\ b\ d'\ h + 1152\ d'^2\ h + 192\ b\ h^2 + 448\ d'\ h^2 + 32\ h^3)\ vc \quad (23)$$

$$E = (-72\ b - 564\ d' - 162\ h)\ Mx + (-162\ b - 564\ d' - 72\ h)\ My + (-12\ b^2 - 174\ b\ d' - 380\ d'^2 - 39\ b\ h - 174\ d'\ h - 12\ h^2)\ Pu + (32\ b^3\ d' + 208\ b^2\ d'^2 + 320\ b\ d'^3 + 64\ d'4 + 16\ b^3\ h + 176\ b^2\ d'\ h + 448\ b\ d'^2\ h + 320\ d'^3\ h + 36\ b^2\ h^2 + 176\ b\ d'\ h^2 + 208\ d'^2\ h^2 + 16\ b\ h^3 + 32\ d'\ h^3)\ vc \quad (24)$$

$$F = (-84\ b\ d' - 288\ d'^2 - 18\ b\ h - 180\ d'\ h - 18\ h^2)\ Mx + (-18\ b^2 - 180\ b\ d' - 288\ d'^2 - 18\ b\ h - 84\ d'\ h)\ My + (-14\ b^2\ d' - 92\ b\ d'^2 - 128\ d'^3 - 3\ b^2\ h - 44\ b\ d'\ h - 92\ d'^2\ h - 3\ b\ h^2 - 14\ d'\ h^2)\ Pu + (8\ b^3\ d'^2 + 32\ b^2\ d'^3 + 32\ b\ d'^4 + 8\ b^3\ d'\ h + 40\ b^2\ d'^2\ h + 64\ b\ d'^3\ h + 32\ d'^4\ h + 2\ b^3\ h^2 + 16\ b^2\ d'\ h^2 + 40\ b\ d'^2\ h^2 + 32\ d'^3\ h^2 + 2\ b^2\ h^3 + 8\ b\ d'\ h^3 + 8\ d'^2\ h^3)\ vc \quad (25)$$

$$G = (-24\ b\ d'^2 - 48\ d'^3 - 12\ b\ d'\ h - 48\ d'^2\ h - 12\ d'\ h^2)\ Mx + (-12\ b^2\ d' - 48\ b\ d'^2 - 48\ d'^3 - 12\ b\ d'\ h - 24\ d'^2\ h)\ My + (-4\ b^2\ d'^2 - 16\ b\ d'^3 - 16\ d'^4 - 2\ b^2\ d'\ h - 12\ b\ d'^2\ h - 16\ d'^3\ h - 2\ b\ d'\ h^2 - 4\ d'^2\ h^2)\ Pu \quad (26)$$

Dimension of Circular Footing

Dimension of footing are *Dp, d*, dimension of column *dc,* Axial load and biaxial bending moment, *Pu, Mx, My* (fig. 6,7).

$$M_z = \sqrt{M_x^2 + M_y^2} \quad (27)$$

$$e_z = \frac{M_z}{P_u} \quad (28)$$

P_u

$$Dp = dc + 4d + 2\,d' \quad (30)$$

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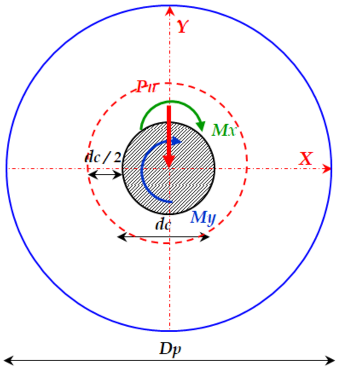

Figure 6. Axial Load Pu, Moment Mx dan My

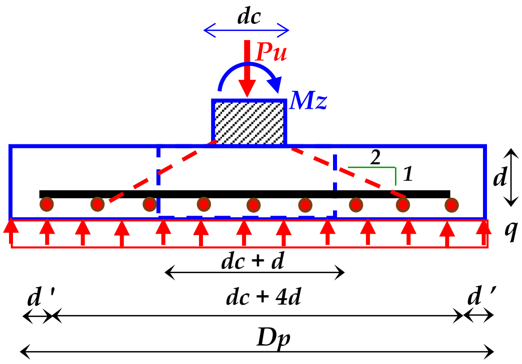

Figure 7. Axial Load Pu, Mz

$$A_c = \frac{\pi}{4} h_c^{\ 2}$$

$$s_k = \frac{\pi}{2} h_c$$

$$A_c = \frac{\pi}{4} Dp^2$$

$$A_p = \frac{\pi}{4} (h_c + 4d + 2d')^2$$

$$q = \frac{P_u}{A_p} \pm \frac{M_z}{W_z}$$

$$W_z = \frac{\pi \ D_p^{\ 3}}{32}$$

$$W_z = \frac{A_p \, D_p}{8}$$

$$d^2 \left(4v_c + \frac{P_u}{A_p} + \frac{M_z}{W_z} \right) + d \left(2v_c + \frac{P_u}{A_p} + \frac{M_z}{W_z} \right) s_k - (A_p - A_c) \left(\frac{P_u}{A_p} + \frac{M_z}{W_z} \right) = 0$$

And subtitute (35.b) to (36),

$$d^2\left(4v_c+\frac{P_u}{A_p}+\frac{8\,M_z}{A_p\,D_p}\right)+d\left(2v_c+\frac{P_u}{A_p}+\frac{8\,M_z}{A_p\,D_p}\right)\left(\frac{\pi}{2}h_c\right)$$
$$-\left(A_p-A_c\right)\left(\frac{P_u}{A_p}+\frac{8M_z}{A_p\,D_p}\right)=0$$

Subtitute (28), (30) to (37),

$$Kd^5+Ld^4+Md^3+Nd^2+Pd+Q=0 \qquad (38)$$

$$K = 256\,\pi\,vc \qquad (39)$$

$$L = (192\,dc\,\pi + 384\,d'\,\pi + 64\,dc\,\pi^2)\,vc \qquad (40)$$

$$M = (16 - 64\,\pi)\,Pu + (48\,dc^2\,\pi + 192\,dc\,d'\,\pi + 192\,d'^2\,\pi + 48\,dc^2\,\pi^2 + 96\,dc\,d'\,\pi^2)\,vc \qquad (41)$$

$$N = 32\,Mz - 128\,Mz\,\pi + (4\,dc + 8\,d' - 40\,dc\,\pi - 96\,d'\,\pi)\,Pu$$
$$+(4\,dc^3\,\pi+24\,dc^2\,d'\,\pi+48\,dc\,d'^2\,\pi+32\,d'^3\,\pi+12\,dc^3\,\pi^2+48\,dc^2\,d'\,\pi^2+48\,dc\,d'^2\,\pi^2)\,vc \qquad (42)$$

$$P = -48\,dc\,Mz\,\pi - 128\,d'\,Mz\,\pi + (-6\,dc^2\,\pi - 44\,dc\,d'\,\pi - 48\,d'^2\,\pi)\,Pu$$
$$+(dc^4\,\pi^2 + 6\,dc^3\,d'\,\pi^2 + 12\,dc^2\,d'^2\,\pi^2 + 8\,dc\,d'^3\,\pi^2)\,vc \qquad (43)$$

$$Q = -32\,dc\,d'\,Mz\,\pi - 32\,d'^2\,Mz\,\pi + (-4\,dc^2\,d'\,\pi - 12\,dc\,d'^2\,\pi - 8\,d'^3\,\pi)\,Pu \qquad (44)$$

RESULT AND DISCUSSION

The results and discussion in the research include the studies that have been described in the research methodology.

Rectangular footing

If known,

fc'	=	30	Mpa
fy	=	300	Mpa
bc	=	400	mm
hc	=	500	mm
Pu	=	1000	kN = 1000×10^3 N
Mx	=	500	kNm = 500×10^6 Nmm
My	=	400	kNm = 400×10^6 Nmm
Ø	=	0.75	

Substitutions all variable to Eq (20) to Eq (26)

A = 1402.17
B = $1.54239*10^6$
C = $3.91853*10^8$
D = $-3.47102*10^{11}$
E = $-1.66428*10^{14}$
F = $-2.24892*10^{16}$
G = $-8.7120*10^{17}$

Result

d1 = -576.841 - 417.88 ***i***
d2 = -576.841 + 417.88 ***i***
d3 = -220.267
d4 = -163.773
d5 = -67.2284

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d6 = 504.95
d = d6 = 504.95 mm ≈ 550 mm

Dimension of rectangular footing are:

h = d + d'= 550 + 100 = 650 mm
Lp = hc + 4d + 2*d'
= 500 + 4(550) + 2(100)
= 2900 mm
Bp = bc + 4d + 2*d'
= 400 + 4(550) + 2(100)
= 2800 mm

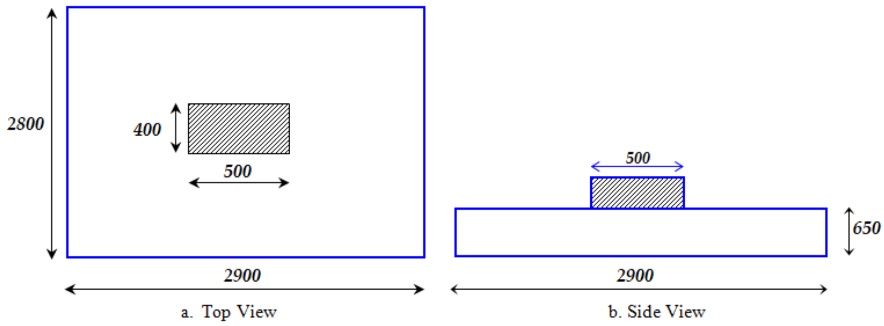

Figure 8. Dimension of regtangular footing

Cicular footing

If known,

fc' = 30 Mpa
fy = 300 Mpa
dc = 500 mm
Pu = 1000 kN = 1000×10^3 N
Mx = 500 kNm = 500×10^6 Nmm
My = 400 kNm = 400×10^6 Nmm
Ø = 0.75

Define: dimension of footing

vc = Ø/3√fc' = 0.75/3√25 = 1.25
d' = 100 mm
Mz = $\sqrt{(500^2 + 400^2)}$ = 640.3124 kNm

Substitutions all variable to Eq (39) to Eq (40)

K = 1101.26
L = $1.01063*10^6$
M = $1.4316*10^8$
N = $-2.81551*10^{11}$
P = $-8.4841*10^{13}$
Q = $-4.39006*10^{15}$

Result

d1 = -553.603 - 417.673 ***i***
d2 = -553.603 + 417.673 ***i***
d3 = -246.509
d4 = -66.8656
d5 = 502.881

d = d5 = 502.881 ≈ 550 mm

Dimension of circular footing are:

h = d + d'= 550 + 100 = 650 mm
Dp = dc + 4d + 2*d'
= 500 + 4(550) + 2(100)
= 2900 mm

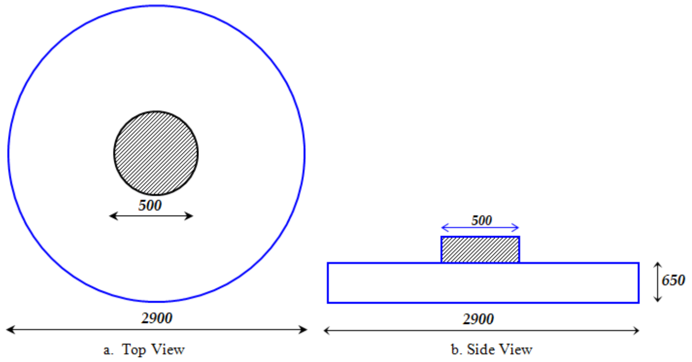

Figure 9. Dimension of circular footing

CONCLUSION

Based on the analytical and numerical evaluation conducted in this study, several conclusions can be drawn as follows: a) the determination of footing dimensions subjected to axial load and bending moment can be formulated analytically by transforming the governing equations into higher-order polynomial equations, b) for rectangular footings, the design equation can be expressed as a sextic polynomial equation involving coefficients A, B, C, D, E, F, and G. The effective depth obtained from the solution of the sextic equation is d = 504.95 mm, which is rounded to 550 mm for practical design considerations, c) based on the calculated effective depth, the resulting rectangular footing dimensions are h = 650 mm, Lp = 2900 mm, and Bp = 2800 mm, while satisfying the detailing requirement d′ > 75 mm, d) for circular footings, the governing design equation can be formulated as a quintic polynomial equation with coefficients K, L, M, N, P, and Q. The numerical solution yields an effective depth of d = 502.881 mm, which is also approximated to 550 mm, e) using this effective depth, the circular footing dimensions are obtained as h = 650 mm and Dp = 2900 mm, which also satisfy the required detailing constraints.

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